School of Engineering
Institute of Materials
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Polymer Science 2024/25

Exercise 7

1. Let us try to better understand the simplest phenomenological mechanical models for viscoelasticity under different types of loading. Schematically draw the evolution of strain or stress over time for the Maxwell model and the Voigt model in case of

   i) a stress relaxation experiment under tension ($\varepsilon = \varepsilon_0$, $\mathrm{d}\varepsilon/\mathrm{d}t = 0$).

   Tip 1: for the Maxwell model, see the Slide 249;

   ii) a creep experiment under tension ($\sigma = \sigma_0$, $\mathrm{d}\sigma/\mathrm{d}t = 0$),

   Tip 2: for the Maxwell model, use the condition $\varepsilon_{dashpot}(t = 0) = 0$ to find an expression for ε that is independent of ε_0.

   Tip 3: The differential equation $y'(x) + \frac{a}{b}y(x) - \frac{c}{b} = 0$ has the solution

$$y = \frac{c}{a}\left[1 - \exp\left(\frac{-ax}{b}\right)\right]$$

   Interpret your results (graphically depicted on Slide 254)!

2. According to the Voigt model for a viscoelastic solid, the creep compliance ($\sigma = \sigma_0$ = constant) is

$$D(t) = \frac{\varepsilon(t)}{\sigma} = \frac{1}{E}\left(1 - exp\left(-\frac{t}{\tau}\right)\right) \quad (1).$$

   where $\tau = \eta/E$ is the *relaxation time*. Equation 1 implies that most of the strain will occur when the time, t, is close to τ.

   What will be the behavior of the Voigt solid in creep within the limits

   - $t << \tau$;
   - $t >> \tau$?

3. A viscoelastic polymer is deformed by a sinusoidal stress, oscillating at an angular frequency ω. Assuming that the variation of the strain, ε, and the stress, σ, can be represented by the equations:

$$\varepsilon = \epsilon_0 \sin(\omega t)$$
$$\sigma = \sigma_0 \sin(\omega t + \delta)$$

where δ is the phase shift between stress and strain, show that the energy dissipated per deformation cycle, ΔU, is given by

$$\Delta U = \sigma_0 \epsilon_0 \pi \sin(\delta)$$

Tips: $\frac{\text{work}}{\text{unit volume}} = \Delta U = \int \sigma \, d\varepsilon$ $\quad cos^2 x = \frac{1+\cos 2x}{2}$

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